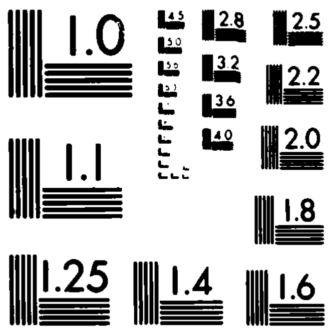
AD-A130 511 TRANSONIC SMALL-DISTURBANCE THEORY FOR DUSTY GASES(U) 1/1
WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER
D A DREW ET AL. MAY 83 MRC-TSR-2521 DAAG29-80-C-0041
UNCLASSIFIED F/G 12/1 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(2)

MRC Technical Summary Report #2521

TRANSONIC SMALL-DISTURBANCE THEORY FOR DUSTY GASES

Donald A. Drew and Fredrick J. Zeigler

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53706

May 1983

(Received March 8, 1983)

Approved for public release
Distribution unlimited

Sponsored by

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

UNIVERSITY OF WISCONSIN-MADISON
MATHEMATICS RESEARCH CENTER

TRANSONIC SMALL-DISTURBANCE THEORY FOR DUSTY GASES

Donald A. Drew* and Fredrick J. Zeigler**

Technical Summary Report #2521
May 1983

ABSTRACT

The correction to transonic small-disturbance theory induced by the addition of contaminant particles (e.g., dust) to the gas is analyzed. Two-phase flow equations governing the particles and inviscid fluid are used. The dusty gas version, a particular limit of the flow equations in which the volumetric concentration of the particles is small, but the mass concentrations of particles and fluid are comparable, is employed. In this setting, a model in which the gas and particles may be viewed as an equivalent gas with modified properties, is derived. This "generalized gas" model behaves like a normal gas with a modified value of γ (the ratio of specific heats). Using this model, a simple method of analyzing transonic small-disturbance theory, by employing a modified transonic similarity parameter, is used to account for the effect of the dust.

AMS (MOS) Subject Classifications: 76T05

Key Words: Two-phase flow, dusty gas, transonic small-disturbance, similarity

Work Unit Number 2 (Physical Mathematics)

*Department of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, NY 12181
**General Dynamics Corporation, Fort Worth Division, P. O. Box 748, Mail Zone 2882, Ft. Worth, TX 76101

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041

SIGNIFICANCE AND EXPLANATION

Many fluid flow problems of interest concern the behavior of a gas which has been contaminated with small particles of dust. The presence of the dust can cause significant changes in the flow, and it is important to analyze an explanation for this phenomenon. This is done by examining a model in which the gas and dust exchange heat and momentum. In the limit of low volumetric concentrations of dust, but with strong coupling between the phases, the model equations are closely approximated by the equations for an adiabatic ideal gas, with modified values of the density and ratio of specific heats. By the use of similarity transformations of these equations, it is possible to relate solutions of flow problems for a gas with dust to solutions of corresponding problems for a clear gas, thus giving an explicit way of calculating the effect of the dust on the flow. Because of their simple form, the equations of transonic flow are used to provide an example of this procedure. It is found that the transonic flow around a thin airfoil for a gas with dust is equivalent to the flow around an airfoil with modified thickness, at a different free-stream mach number.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

TRANSONIC SMALL-DISTURBANCE THEORY FOR DUSTY GASES

Donald A. Drew* and Fredrick J. Zeigler**

1. INTRODUCTION

The design of machinery which utilizes or involves the flow of a gas is important in many practical situations. Often such machines must be operated in an environment where the gas is contaminated with small particles (dust) which may degrade performance, cause wear or necessitate filtering, all with undesirable economic consequences.

In order to better understand the flow of both the dust and gas, we examine a model in which the gas and dust exchange heat and momentum. The equations of conservation of mass, momentum and energy for each material are simplified by assuming low volumetric concentrations of a relatively heavy dust. This set of dusty gas equations is then further approximated by assuming strong coupling between the materials. The resulting system of equations is analogous to the equations for the adiabatic motion of an ideal gas, except that the gas constant γ and the density ρ_m are modified to reflect the heat capacity and the density of the dust. We further show that this generalized gas supports discontinuities in properties (generalized schocks) which consist of a shock in the gas, followed by a relaxation back to equilibrium (both thermal and mechanical) of the dust particles.

We note that the addition of a moderate amount of dust (0.1% by volume) can increase the Mach number of a flow by an appreciable amount (~ 25%). This could have serious consequences in operation and/or efficiency in many devices.

Finally, we discuss the transonic small-disturbance theory. It is shown that the effect of the dust is to modify the flow in such a way that it is equivalent to the flow of a clear gas at a different Mach number around an airfoil of a different thickness. This suggests a way to study the behavior of airfoils of different thicknesses (but similar shapes) may be to add dust to the flow.

*Department of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, NY 12181
**General Dynamics Corporation, Fort Worth Division, P. O. Box 748, Mail Zone 2882, Ft. Worth, TX 76101

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041.

2. EQUATIONS OF TWO-PHASE FLOW

The fluid is assumed to be inviscid, and to contain small particles of 'dust' such that there is no exchange of matter between the phases. Under these conditions, equations governing the flow of particles and fluid are

$$\frac{\partial \alpha\rho_p}{\partial t} + \nabla\cdot\alpha\rho_p\vec{q}_p = 0 \ , \qquad (2.1a)$$

$$\frac{\partial(1-\alpha)\rho_f}{\partial t} + \nabla\cdot(1-\alpha)\rho_f\vec{q}_f = 0 \ , \qquad (2.1b)$$

$$\alpha\rho_p\left(\frac{\partial\vec{q}_p}{\partial t} + \vec{q}_p\cdot\nabla\vec{q}_p\right) = -\alpha\nabla p + \alpha b_M(\vec{q}_f - \vec{q}_p) \ , \qquad (2.2a)$$

$$(1-\alpha)\rho_f\left(\frac{\partial\vec{q}_f}{\partial t} + \vec{q}_f\cdot\nabla\vec{q}_f\right) = -(1-\alpha)\nabla p + \alpha b_M(\vec{q}_p - \vec{q}_f) \ , \qquad (2.2b)$$

$$\alpha\rho_p\left[\frac{\partial\left(\epsilon_p + \frac{1}{2}q_p^2\right)}{\partial t} + \vec{q}_p\cdot\nabla\left(\epsilon_p + \frac{1}{2}q_p^2\right)\right] =$$

$$= -\nabla\alpha\cdot p\vec{q}_p - p\nabla\alpha\cdot\vec{q}_p + \alpha b_M(\vec{q}_f - \vec{q}_p)\cdot\vec{q}_p + \alpha H_M(T_f - T_p) \ , \qquad (2.3a)$$

$$(1-\alpha)\rho_f\left[\frac{\partial\left(\epsilon_f + \frac{1}{2}q_f^2\right)}{\partial t} + \vec{q}_f\cdot\nabla\left(\epsilon_f + \frac{1}{2}q_f^2\right)\right] =$$

$$= -\nabla(1-\alpha)\cdot p\vec{q}_f - p\nabla\alpha\cdot\vec{q}_p - \alpha b_M(\vec{q}_f - \vec{q}_p)\cdot\vec{q}_p + \alpha H_M(T_p - T_f) \ , \qquad (2.3b)$$

where 2.1, 2.2, and 2.3 are equations of conservation of mass, momentum, and energy for the particle and fluid phases. Here $\vec{q}$ represents the velocity (p for particles, f for fluid), α is the dust volumetric density = dust volume/total volume, ρ denotes the phasic density (for the particles, ρ_p = dust mass/dust volume), p is the pressure, and ϵ is the internal energy per unit mass.

In addition, equations of state are needed for both phases. The dust is assumed to be incompressible

$$\rho_p = \text{const} \ ; \qquad (2.4)$$

for the fluid, the ideal gas law

$$p = R'\rho_f T_f \tag{2.5}$$

holds. The particles are assumed to be calorically perfect, so that

$$\varepsilon_p = c_v^{(p)} T_p ,$$

where $c_v^{(p)}$ denotes the constant volume specific heat of the particles; and, for the gas,

$$\varepsilon_f = c_v^{(f)} T_f + p/\rho_f . \tag{2.6}$$

The terms on the right of (2.2) and (2.3) involving b_M and H_M reflect momentum and energy transfer between the particles and gas. For b_M we assume the general form

$$b_M = \frac{3}{8} c_D \rho_f |\vec{q}_p - \vec{q}_f| \cdot \frac{1}{a} \tag{2.7}$$

where c_D is the drag coefficient, and a is the effective particle radius. If the relative velocities between the two phases are small,

$$c_D = 24/Re \tag{2.8}$$

where

$$Re = \frac{2aU}{\nu_f} , \tag{2.9}$$

U being a reference velocity. Then

$$b_M = \frac{9}{2} \rho_f \nu_f \frac{1}{a^2} \tag{2.10}$$

according to the Stokes law.

Expressed in terms of the Nusselt number Nu, the heat transfer coefficient H_M is

$$H_M = \frac{3}{2} \kappa \; Nu \cdot \frac{1}{a^2} \tag{2.11}$$

where κ is the coefficient of heat conductivity. By including the Prandtl number $P_Y = \mu_f/\kappa$, this may be written

$$H_M = \frac{1}{3} Nu \; Pr \; b_M . \tag{2.12}$$

3.

The dusty gas limit of the previous two-phase flow equations corresponds to the limit $\alpha << 1$, $\rho_f << \rho_p$ and $\alpha\rho_p \sim \rho_f$. In order to exhibit this limit explicitly we first scale the variables and put the equations into nondimensional form.

Thus, we assume the problem contains a typical length scale L, velocity scale U, gas density scale Γ, temperature scale T_0, and volumetric concentration scale A. We then set

$$\hat{\vec{x}} = \vec{x}/L \tag{3.1a}$$

$$\hat{t} = tU/L \tag{3.1b}$$

$$\alpha(\vec{x},t) = A\hat{\alpha}(\hat{\vec{x}},\hat{t}) \tag{3.2a}$$

$$\rho_f(\vec{x},t) = \Gamma\hat{\rho}_f(\hat{\vec{x}},\hat{t}) \tag{3.2b}$$

$$\vec{q}_p(\vec{x},t) = U\hat{\vec{q}}_p(\hat{\vec{x}},\hat{t}) \tag{3.2c}$$

$$\vec{q}_f(\vec{x},t) = U\hat{\vec{q}}_f(\hat{\vec{x}},\hat{t}) \tag{3.2d}$$

$$p(\vec{x},t) = \Gamma U^2\hat{p}(\hat{\vec{x}},\hat{t}) \tag{3.2e}$$

$$\varepsilon_p(\vec{x},t) = U^2\hat{\varepsilon}_p(\hat{\vec{x}},\hat{t}) \tag{3.2f}$$

$$\varepsilon_f(\vec{x},t) = U^2\hat{\varepsilon}_f(\hat{\vec{x}},\hat{t}) \tag{3.2g}$$

$$T_p(\vec{x},t) = T_0\hat{T}_p(\hat{\vec{x}},\hat{t}) \tag{3.2h}$$

$$T_f(\vec{x},t) = T_0\hat{T}_f(\hat{\vec{x}},\hat{t}) \ . \tag{3.2i}$$

The following dimensionless combinations will be used:

$$s = \Gamma/\rho_p \tag{3.3a}$$

$$\varepsilon = \Gamma U/b_M AL \tag{3.3b}$$

$$h = H_M T_0/b_M U^2 \tag{3.3c}$$

$$\gamma = R' T_0 / U^2 \tag{3.3d}$$

$$c = c_v^{(p)} T_0 / U^2 \tag{3.3e}$$

$$\hat{c} = c_v^{(f)} T_0 / U^2 \ . \tag{3.3f}$$

The dimensionless two-phase flow equations are

$$\frac{\partial \hat{\alpha}}{\partial \hat{t}} + \hat{\nabla} \cdot (\hat{\alpha} \hat{\vec{q}}_p) = 0 \tag{3.4a}$$

$$\frac{\partial (1 - A\hat{\alpha})\hat{\rho}_f}{\partial \hat{t}} + \hat{\nabla} \cdot (1 - A\hat{\alpha})\hat{\rho}_f \hat{\vec{q}}_f = 0 \tag{3.4b}$$

$$\frac{\partial \hat{\vec{q}}_p}{\partial \hat{t}} + \hat{\vec{q}}_p \cdot \hat{\nabla} \hat{\vec{q}}_p = -s\hat{\nabla}\hat{p} + \frac{s}{\varepsilon A} (\hat{\vec{q}}_f - \hat{\vec{q}}_p) \tag{3.5a}$$

$$(1 - A\hat{\alpha})\hat{\rho}_f \left(\frac{\partial \hat{\vec{q}}_f}{\partial \hat{t}} + \hat{\vec{q}}_f \cdot \hat{\nabla} \hat{\vec{q}}_f \right) = -(1 - A\hat{\alpha})\hat{\nabla}\hat{p} + \frac{1}{\varepsilon} \hat{\alpha} (\hat{\vec{q}}_p - \hat{\vec{q}}_f) \tag{3.5b}$$

$$\hat{\alpha}\left[\frac{\partial (\hat{\varepsilon}_p + \frac{1}{2} \hat{q}_p^2)}{\partial \hat{t}} + \hat{\vec{q}}_p \cdot \hat{\nabla} (\hat{\varepsilon}_p + \frac{1}{2} \hat{q}_p^2) \right] =$$

$$= -s\hat{\nabla}\hat{\alpha} \cdot \hat{p}\hat{\vec{q}}_p - s\hat{p}\hat{\nabla}\hat{\alpha} \cdot \hat{\vec{q}} + \frac{s}{\varepsilon A} \hat{\alpha} (\hat{\vec{q}}_f - \hat{\vec{q}}_p) \cdot \hat{\vec{q}}_p + \frac{hs}{\varepsilon A} \hat{\alpha} (\hat{T}_f - \hat{T}_p) \tag{3.6a}$$

$$(1 - A\hat{\alpha})\hat{\rho}_f \left[\frac{\partial (\hat{\varepsilon}_f + \frac{1}{2} \hat{q}_f^2)}{\partial \hat{t}} + \hat{\vec{q}}_f \cdot \hat{\nabla} (\hat{\varepsilon}_f + \frac{1}{2} \hat{q}_f^2) \right] =$$

$$= -\hat{\nabla}(1 - A\hat{\alpha}) \cdot \hat{p}\hat{\vec{q}}_f - A\hat{p}\hat{\nabla}\hat{\alpha} \cdot \hat{\vec{q}}_p - \frac{1}{\varepsilon} \hat{\alpha} (\hat{\vec{q}}_f - \hat{\vec{q}}_p) \cdot \hat{\vec{q}}_p + \frac{h}{\varepsilon} \hat{\alpha} (\hat{T}_p - \hat{T}_f) \ . \tag{3.6b}$$

The dusty gas limit corresponds to $A \to 0$, $s \to 0$ (since $\rho_f \ll \rho_p$ = const implies $\Gamma/\rho_p = s \ll 1$), with $f = A/S$ remaining fixed. The dusty gas formulation is

$$\frac{\partial \hat{\alpha}}{\partial \hat{t}} + \hat{\nabla} \cdot (\hat{\alpha} \hat{\vec{q}}_p) = 0 \tag{3.7a}$$

$$\frac{\partial \hat{\rho}_f}{\partial \hat{t}} + \hat{\nabla} \cdot \hat{\rho}_f \hat{\vec{q}}_f = 0 \tag{3.7b}$$

$$\frac{\partial \hat{\vec{q}}_p}{\partial \hat{t}} + \hat{\vec{q}}_p \cdot \hat{\nabla} \hat{\vec{q}}_p = \frac{1}{\varepsilon f} (\hat{\vec{q}}_f - \hat{\vec{q}}_p) \tag{3.8a}$$

$$\hat{\rho}_f \left(\frac{\partial \hat{\vec{q}}_f}{\partial \hat{t}} + \hat{\vec{q}}_f \cdot \hat{\nabla} \hat{\vec{q}}_f \right) = -\hat{\nabla}\hat{p} + \frac{1}{\varepsilon} \hat{\alpha} (\hat{\vec{q}}_p - \hat{\vec{q}}_f) \tag{3.8b}$$

$$\left[\frac{\partial (\hat{\varepsilon}_p + \frac{1}{2} \hat{q}_p^2)}{\partial \hat{t}} + \hat{\vec{q}}_p \cdot \hat{\nabla} (\hat{\varepsilon}_p + \frac{1}{2} \hat{q}_p^2) \right] = \frac{1}{\varepsilon f} (\hat{\vec{q}}_f - \hat{\vec{q}}_p) \cdot \hat{\vec{q}}_p + \frac{h}{\varepsilon f} (\hat{T}_f - \hat{T}_p) = 0 \tag{3.9a}$$

$$\hat{\rho}_f \left[\frac{\partial (\hat{\varepsilon}_f + \frac{1}{2} q_f^2)}{\partial \hat{t}} + \hat{\vec{q}}_f \cdot \hat{\nabla} (\hat{\varepsilon}_f + \frac{1}{2} \hat{q}_f^2) \right] =$$

$$= -\hat{\nabla} \cdot \hat{p} \hat{\vec{q}}_f - \frac{1}{\varepsilon} \hat{\alpha} (\hat{\vec{q}}_f - \hat{\vec{q}}_p) \cdot \hat{\vec{q}}_p + \frac{h}{\varepsilon} \hat{\alpha} (\hat{T}_p - \hat{T}_f) \ . \tag{3.9b}$$

The equations of state take the form

$$\hat{p} = r\hat{\rho}_f \hat{T}_f \ , \tag{3.10}$$

$$\hat{\varepsilon}_p = c\hat{T}_p \ , \tag{3.11}$$

$$\hat{\varepsilon}_f = \hat{c}\hat{T}_f + \hat{p}/\hat{\rho}_f \ . \tag{3.12}$$

4. GENERALIZED GAS MODEL

Let us see the effect which the effective particle radius, a, has on the dusty gas equations. By using the Stokes drag formula, we may write

$$\varepsilon = \frac{2}{9}\frac{Ua^2}{\gamma_f AL}\frac{1}{\hat{\rho}_f} = \left(\frac{Ua}{\gamma_f}\right)\left(\frac{a}{L}\right)\left(\frac{1}{A}\right)\left(\frac{2}{9\hat{\rho}_f}\right) . \tag{4.1}$$

Thus, for sufficiently small particles, ε will be small. In fact, for many flows Re a and a/L are small, but A is also small. The rest of our discussion will be limited to the regime $\varepsilon \ll 1$.

For this case, inspection of equation 3.5 shows that $\vec{q}_f = \vec{q}_p$ (we now drop the carets for convenience) unless the accelerations are large. This suggest a model which we term the generalized gas model, which has the property that $\vec{q}_p = \vec{q}_f + O(\varepsilon)$, except in places where the flow fields change rapidly. We shall call these regions of rapid change generalized shocks. As we shall see, this model is analogous to a gas with changed properties.

Consider the flow fields α, $\vec{q}_p$, $\vec{q}_\rho$, etc. as functions of $\vec{x}$, t, and ε. In the generalized gas model, we consider a limit of the dusty gas equations in which $\varepsilon \to 0$, with $\vec{x}$, t held fixed. Away from generalized shocks, we therefore consider an expansion

$$\alpha(x,t;\varepsilon) = \alpha^{(0)}(x,t) + \varepsilon\alpha^{(1)}(x,t) + \ldots \tag{4.2}$$

with similar expressions for the other flow quantities.

Substituting these expressions into the dusty gas equations and equating terms of equal order to zero gives the following:

$$\vec{q}_f^{\,(0)} = \vec{q}_p^{\,(0)} \equiv \vec{q} \tag{4.3}$$

$$T_f^{(0)} = T_p^{(0)} \equiv T \tag{4.4}$$

$$\frac{\partial\alpha^{(0)}}{\partial t} + \nabla\cdot\alpha^{(0)}\vec{q} = 0 \tag{4.5}$$

$$\frac{\partial\rho_f^{(0)}}{\partial t} + \nabla\cdot\rho_f^{(0)}\vec{q} = 0 \tag{4.6}$$

$$f\left(\frac{\partial \vec{q}}{\partial t} + \vec{q}\cdot\nabla\vec{q}\right) = \vec{q}_f^{(1)} - \vec{q}_p^{(1)} \tag{4.7}$$

$$\rho_f^{(0)}\left(\frac{\partial \vec{q}}{\partial t} + \vec{q}\cdot\nabla\vec{q}\right) = -\nabla p^{(0)} + \alpha^{(0)}(\vec{q}_p^{(1)} - \vec{q}_f^{(1)}) \tag{4.8}$$

$$f\left[\frac{\partial(cT + \frac{1}{2}q^2)}{\partial t} + \vec{q}\cdot\nabla(cT + \frac{1}{2}q^2)\right] = (\vec{q}_f^{(1)} - \vec{q}_p^{(1)})\cdot\vec{q} + h(T_f^{(1)} - T_p^{(1)}) \tag{4.9}$$

$$\rho_f^{(0)}\left[\frac{\partial(\varepsilon_f^{(0)} + \frac{1}{2}q^2)}{\partial t} + \vec{q}\cdot\nabla(\varepsilon_f^{(0)} + \frac{1}{2}q^2)\right] =$$

$$= -\nabla p^{(0)}\vec{q} - \alpha^{(0)}(\vec{q}_f^{(1)} - \vec{q}_p^{(1)})\cdot\vec{q} + h\alpha^{(0)}(T_p^{(1)} - T_f^{(1)})$$

$$p^{(0)} = r\rho_f^{(0)}T \tag{4.11}$$

$$\varepsilon_f^{(0)} = \hat{c}T + p^{(0)}/\rho_f^{(0)} \ . \tag{4.12}$$

This model allows us to derive the equations needed at the lowest order. Adding $\alpha^{(0)}$ times (4.7) to (4.8) yields

$$(\rho_f^{(0)} + \alpha^{(0)}f)\left(\frac{\partial \vec{q}}{\partial t} + \vec{q}\cdot\nabla\vec{q}\right) = -\nabla p^{(0)} \tag{4.13}$$

A similar combination of (4.9) and (4.10) gives

$$\rho_f^{(0)}\left[\frac{\partial \hat{c}T}{\partial t} + \vec{q}\cdot\nabla\hat{c}T\right] + \alpha^{(0)}f\left[\frac{\partial cT}{\partial t} + \vec{q}\cdot\nabla cT\right] +$$

$$+ (\rho_f^{(0)} + \alpha^{(0)}f)\left[\frac{\partial(\frac{1}{2}q^2)}{\partial t} + \vec{q}\cdot\nabla(\frac{1}{2}q^2)\right] +$$

$$+ \rho_f^{(0)}\left[\frac{\partial(p^{(0)}/\rho_f^{(0)})}{\partial t} + \vec{q}\cdot\nabla(p^{(0)}/\rho_f^{(0)})\right] = -\nabla\cdot p^{(0)}\vec{q} \ . \tag{4.14}$$

Additionally, (4.5) and (4.6) may be combined as

$$\frac{\partial(\rho_f^{(0)} + \alpha^{(0)}f)}{\partial t} + \nabla\cdot(\rho_f^{(0)} + \alpha^{(0)}f)\vec{q} = 0 = \frac{\partial\rho_m}{\partial t} + \nabla\cdot\rho_m\vec{q} \tag{4.15}$$

where $\rho_m = \rho_f^{(0)} + \alpha^{(0)}f$ is the mixture density. Using (4.5) and (4.6) we obtain

$$\frac{\partial}{\partial t} \ln\left(\frac{\alpha^{(0)}}{\rho_f^{(0)}}\right) + \vec{q}\cdot\nabla \ln(\alpha^0/\rho_f^{(0)}) = 0 \ . \tag{4.16}$$

Therefore $\alpha^0/\rho_f^{(0)}$ is constant for a fluid particle.

The energy equation may be rewritten in order to yield further information. First, taking the dot product of $\vec{q}$ with (4.13) shows that

$$(\rho_f^{(0)} + \alpha^{(0)}f)\left[\frac{\partial(\frac{1}{2}q^2)}{\partial t} + \vec{q}\cdot\nabla(\tfrac{1}{2}q^2)\right] = -\vec{q}\cdot\nabla p^{(0)} \ . \tag{4.17}$$

Then, from (4.14), we are left with

$$(\rho_f^{(0)}\hat{c} + \alpha^{(0)}fc)\left[\frac{\partial T}{\partial t} + \vec{q}\cdot\nabla T\right] + \rho_f^{(0)}\left[\frac{\partial(p^{(0)}/\rho_f^{(0)})}{\partial t} + \vec{q}\cdot\nabla(p^{(0)}/\rho_f^{(0)})\right] = -p^{(0)}\nabla\cdot\vec{q} \ . \tag{4.18}$$

With the expressions (4.6) and (4.11), this yields

$$(\rho_f^{(0)}\hat{c} + \alpha^{(0)}fc + \rho_f^{(0)}r)\left[\frac{\partial T}{\partial t} + \vec{q}\cdot\nabla T\right] = rT\left[\frac{\partial\rho_f^{(0)}}{\partial t} + \vec{q}\cdot\nabla\rho_f^{(0)}\right] \tag{4.19}$$

Therefore, following a fluid particle,

$$T = \mathrm{Const}\cdot(\rho_f^{(0)})^{\hat{\gamma}-1} \tag{4.20}$$

where

$$\hat{\gamma} = \frac{\hat{c} + \alpha^{(0)}fc/\rho_f^{(0)}}{\hat{c} + r + \alpha^{(0)}fc/\rho_f^{(0)}} = \frac{\gamma + \dfrac{\alpha^{(0)}f}{\rho_f^{(0)}}\dfrac{c}{\hat{c}+r}}{1 + \dfrac{\alpha^{(0)}f}{\rho_f^{(0)}}\dfrac{c}{\hat{c}+r}} \tag{4.21}$$

is constant on each streamline.

Since $\rho_f^{(0)} + \alpha^{(0)}f = \rho_m$, and $\alpha^{(0)}/\rho_f^{(0)}$ is constant following a fluid particle, (4.20) may be written

$$T = \text{Const}\cdot\rho_m^{\hat{\gamma}-1} \tag{4.22}$$

where the constant in (4.22) is $1 + \alpha^{(0)}f/\rho_f^{(0)}$ times the one in (4.20).

We now assume that $\alpha^{(0)}/\rho_f^{(0)}$ is the same for all streamlines at $t = 0$, and that we have a constant value of $\alpha^{(0)}/\rho_f^{(0)}$ far upstream for all time. Then, $\hat{\gamma}$ and the constant in (4.22) are constant for the whole flow field for all time. Dropping the (0) superscript for the pressure, the generalized gas model is

$$\frac{\partial\rho_m}{\partial t} + \nabla\cdot\rho_m\vec{q} = 0 \tag{4.23}$$

$$\rho_m\left(\frac{\partial\vec{q}}{\partial t} + \vec{q}\cdot\nabla\vec{q}\right) = -\nabla p \ , \tag{4.24}$$

$$p/p_0 = (\rho_m/\rho_0)^{\hat{\gamma}} \ , \tag{4.25}$$

where ρ_0 and p_0 are constants. Thus, a dusty gas with small dust particles behaves like *a gas with a modified* γ. We emphasize that this derivation assumes that we are not near a shock.

We note that the speed of sound for a generalized gas is given by

$$a^2 = \frac{dp}{d\rho_m} = \frac{\hat{\gamma}p}{\rho_m} = \frac{\hat{\gamma}\rho_f^{(0)}}{\gamma\rho_m}\left(\frac{\gamma p}{\rho_f^{(0)}}\right) = \frac{\hat{\gamma}\rho_f^{(0)}}{\gamma\rho_m}\hat{a}^2 \ ,$$

where $\hat{a}$ is the speed of sound in the clear gas. Moreover, since $\rho_m = \rho_f^{(0)} + \alpha^{(0)}f$,

$$\frac{\hat{\gamma}\rho_f^{(0)}}{\gamma\rho_m} = \frac{\hat{\gamma}}{\gamma}\,\frac{1}{1 + \dfrac{\alpha^{(0)}f}{\rho_f^{(0)}}}.$$

5. GENERALIZED SHOCK STRUCTURE

In order to derive expressions which measure the correction to transonic small-disturbance theory caused by the addition of dust to the gas, we must examine how shock waves fit into this framework. The generalized gas equations (4.23)-(4.25) are not valid in regions of rapid change. In this section we shall derive equations valid for such regions, for the purpose of obtaining shock relations which can be used with the equations of the previous section. We assume that the conditions for a dusty gas are not violated in the shock, so we shall start the analysis by using (3.9)-(3.12).

We consider only planar shocks, with the generalized shock location given by

$$x = x_s(t) , \tag{5.1}$$

y, z arbitrary.

A coordinate system fixed in the shock will be used, with the positive x-axis in the direction of net flow through the shock. Setting $X = x - x_s(t)$, $Y = y$, $Z = z$, $T = t$ transforms the dusty gas equations to

$$\frac{\partial\alpha}{\partial T} + \frac{\partial(\alpha U_p)}{\partial X} + \frac{\partial(\alpha V_p)}{\partial Y} + \frac{\partial(\alpha W_p)}{\partial Z} = 0 \tag{5.2}$$

$$\frac{\partial\rho_f}{\partial T} + \frac{\partial\rho_f U_f}{\partial X} + \frac{\partial\rho_f V_f}{\partial Y} + \frac{\partial\rho_f W_f}{\partial Z} = 0 \tag{5.3}$$

$$\frac{\partial U_p}{\partial T} + U_p \frac{\partial U_p}{\partial X} + V_p \frac{\partial U_p}{\partial Y} + W_p \frac{\partial U_p}{\partial Z} = \frac{1}{f\varepsilon} (U_f - U_p) - \frac{d^2 x_s}{dt^2} \tag{5.4}$$

$$\frac{\partial V_p}{\partial T} + U_p \frac{\partial V_p}{\partial X} + V_p \frac{\partial V_p}{\partial Y} + W_p \frac{\partial V_p}{\partial Z} = \frac{1}{f\varepsilon} (V_f - V_p) \tag{5.5}$$

$$\frac{\partial W_p}{\partial t} + U_p \frac{\partial W_p}{\partial X} + V_p \frac{\partial W_p}{\partial Y} + W_p \frac{\partial W_p}{\partial Z} = \frac{1}{f\varepsilon} (W_f - W_p) \tag{5.6}$$

$$\rho_f\left(\frac{\partial U_f}{\partial T} + U_f \frac{\partial U_f}{\partial X} + V_f \frac{\partial U_f}{\partial Y} + W_f \frac{\partial U_f}{\partial Z}\right) = \frac{-\partial p}{\partial X} + \frac{\alpha}{\varepsilon} (U_p - U_f) - \rho_f \frac{d^2 x_s}{dt^2} \tag{5.7}$$

$$\rho_f\left(\frac{\partial V_f}{\partial T} + U_f \frac{\partial V_f}{\partial X} + V_f \frac{\partial V_f}{\partial Y} + W_f \frac{\partial V_f}{\partial Z}\right) = \frac{-\partial p}{\partial Y} + \frac{\alpha}{\varepsilon} (V_p - V_f) \tag{5.8}$$

$$\rho_f\left(\frac{\partial W_f}{\partial T} + U_f \frac{\partial W_f}{\partial X} + V_f \frac{\partial W_f}{\partial Y} + W_f \frac{\partial W_f}{\partial Z}\right) = \frac{-\partial p}{\partial Z} + \frac{\alpha}{\varepsilon} (W_p - W_f) \tag{5.9}$$

$$c\left(\frac{\partial T_p}{\partial T} + U_p \frac{\partial T_p}{\partial X} + V_p \frac{\partial T_p}{\partial Y} + W_p \frac{\partial T_p}{\partial Z}\right) = \frac{h}{f\varepsilon} (T_f - T_p) \tag{5.10}$$

$$\rho_f \hat{c}\left(\frac{\partial T_f}{\partial T} + U_f \frac{\partial T_f}{\partial X} + V_f \frac{\partial T_f}{\partial Y} + W_f \frac{\partial T_f}{\partial Z}\right) = -p\left(\frac{\partial U_f}{\partial X} + \frac{\partial V_f}{\partial Y} + \frac{\partial W_f}{\partial Z}\right)$$

$$- \frac{\alpha}{\varepsilon} [(U_f - U_p)^2 + (V_f - V_p)^2 + (W_f - W_p)^2] + \frac{h}{\varepsilon} \alpha(T_p - T_f) \ . \tag{5.11}$$

In the above, $U_{p,f} = u_{p,f} - \frac{dx_s}{dt}$, $V_{p,f} = v_{p,f}$, $W_{p,f} = w_{p,f}$. The choice of coordinate system dictates $U_{p,f} > 0$.

To observe the details of the shock structure, we examine a distinguished limit of these equations most appropriate for regions of rapid change. In order to get the richest equations, we choose $\xi = X/\varepsilon$. Then we have

$$\frac{\partial(\alpha U_p)}{\partial \xi} = 0 \tag{5.12}$$

$$\frac{\partial(\rho_f U_f)}{\partial \xi} = 0 \tag{5.13}$$

$$U_p \frac{\partial U_p}{\partial \xi} = \frac{1}{f} (U_f - U_p) \tag{5.14}$$

$$U_p \frac{\partial V_p}{\partial \xi} = \frac{1}{f} (V_f - V_p) \tag{5.15}$$

$$U_p \frac{\partial W_p}{\partial \xi} = \frac{1}{f} (W_f - W_p) \tag{5.16}$$

$$\rho_f U_f \frac{\partial U_f}{\partial \xi} = \frac{-\partial p}{\partial \xi} + \alpha(U_p - U_f) \tag{5.17}$$

$$\rho_f U_f \frac{\partial V_f}{\partial \xi} = \alpha(V_p - V_f) \tag{5.18}$$

$$\rho_f U_f \frac{\partial W_f}{\partial \xi} = \alpha(W_p - W_f) \tag{5.19}$$

$$cU_p \frac{\partial T_p}{\partial \xi} = \frac{h}{f}(T_f - T_p) \tag{5.20}$$

$$\rho_f \hat{c} U_f \frac{\partial T_f}{\partial \xi} = -p \frac{\partial U_f}{\partial \xi} - \alpha[(U_f - U_p)^2 + (V_f + V_p)^2 + (W_f - W_p)^2] + h\alpha(T_p - T_f) \; . \tag{5.21}$$

Boundary conditions are given as:

$$U_f, U_p \to u^{\pm} - \frac{dx_s}{dt}; \quad V_f, V_p \to v^{\pm}; \quad W_f, W_p \to w^{\pm};$$
$$T_f, T_p \to T^{\pm}; \quad \alpha \to \alpha^{\pm}; \quad \rho_f \to \rho_f^{\pm} \quad \text{as} \quad \xi \to \pm\infty \; .$$

Let us consider the possibility that these equations have a shock solution, that is, a discontinuous solution which satisfies appropriate jump conditions. Equations (5.12)-(5.20) give rise to the jump conditions

$$[\alpha U_p] = 0 \tag{5.22}$$

$$[\rho_f U_f] = 0 \tag{5.23}$$

$$[U_p^2] = 0 \tag{5.24}$$

$$[U_p V_p] = 0 \tag{5.25}$$

$$[U_p W_p] = 0 \tag{5.26}$$

$$[\rho_f U_f^2 + p] = 0 \tag{5.27}$$

$$[\rho_f U_f V_f] = 0 \tag{5.28}$$

$$[\rho_f U_f W_f] = 0 \tag{5.29}$$

$$[cU_p T_p] = 0 \tag{5.30}$$

$$[\rho_f \hat{c} U_f T_f + pU_f] = 0 \tag{5.31}$$

Conditions (5.22), (5.24), (5.25), (5.26), and (5.30) show that the particle properties are continuous across a shock. The other conditions show that the gas properties are not affected by the dust properties in the shock; it is a "classical" gas shock.

Using (5.23) in (5.28), (5.29) easily shows that V_f, W_f are continuous. With $V_{p,f}$ and $W_{p,f}$ continuous across the shock, and being governed by (5.15), (5.16), (5.18), (5.19) in the rest of the transition region, this combined with the boundary conditions shows that $V_p = V_f$ and $W_p = W_f$ throughout the generalized shock.

The remaining differential equations describe the evolution of the gas and particle flow on the length scale ϵ. The smooth evolution must be on the ξ-positive side of the shock, since the solutions will be well-behaved as $\xi \to 0$.

Let us now examine the transition region, the section behind the shock where the discontinuity in U_f has caused $U_p \neq U_f$. We assume that the actual shock is at $\xi = 0$.

From (5.12)

$$\alpha U_p = \phi_p(t) = \alpha^{0+} U_p^{0+} = \alpha^+ \left(u^+ - \frac{dx_s}{dt}\right) . \tag{5.32}$$

But particle properties are continuous, so $\alpha^{0+} = \alpha^{0-}$, $U_p^{0+} = U_p^{0-}$, and

$$\alpha^- \left(u^- - \frac{dx_s}{dt}\right) = \alpha^{0-} U_p^{0-} = \alpha^+ \left(u^+ - \frac{dx_s}{dt}\right) . \tag{5.33}$$

Using (5.13) we may similarly derive

$$\rho_f^+ \left(u^+ - \frac{dx_s}{dt}\right) = \rho_f^- \left(u^- - \frac{dx_s}{dt}\right) . \tag{5.34}$$

Therefore $\alpha^-/\rho_f^- = \alpha^+/\rho_f^+$, and

$$\rho_m^- \left(u^- - \frac{dx_s}{dt}\right) = \rho_m^+ \left(u^+ - \frac{dx_s}{dt}\right) . \tag{5.35}$$

To get the total momentum, we add αf times (5.14) to (5.17), which yields

$$\alpha f U_p^2 + \rho_f U_f^2 + p = \mu(t) . \tag{5.36}$$

By an argument similar to the one leading to (5.35) we find

$$\rho_m^-\left(u^- - \frac{dx_s}{dt}\right)^2 + p^- = \rho_m^+\left(u^+ - \frac{dx_s}{dt}\right)^2 + p^+ . \tag{5.37}$$

By taking a linear combination of (5.20) and (5.21), and using (5.14) and (5.17) on the resulting expression, we may derive the energy equation

$$\alpha f c U_p T_p + \rho_f \hat{c} U_f T_f + \alpha f U_p^3/2 + \rho_f U_f^3/2 + pU_f = e(t) . \tag{5.38}$$

The corresponding jump condition is

$$\left(\alpha^- f c + \rho_f^- \hat{c}\right)\left(u^- - \frac{dx_s}{dt}\right)T^- + \frac{1}{2}\rho_m^-\left(u^- - \frac{dx_s}{dt}\right)^3 + p^-\left(u^- - \frac{dx_s}{dt}\right)$$

$$= \left(\alpha^+ f c + \rho_f^+ \hat{c}\right)\left(u^+ - \frac{dx_s}{dt}\right)T^+ + \frac{1}{2}\rho_m^+\left(u^+ - \frac{dx_s}{dt}\right)^3 + p^+\left(u^+ - \frac{dx_s}{dt}\right) . \tag{5.39}$$

Two equations cannot be explicitly integrated across the generalized shock. They can be taken to be

$$U_p \frac{dU_p}{d\xi} = \frac{1}{f}\left(U_f - U_p\right) , \tag{5.40}$$

$$cU_p \frac{dT_p}{d\xi} = \frac{h}{f}\left(T_f - T_p\right) . \tag{5.41}$$

We note that (5.40) and (5.41) govern the non-equilibrium evolution of the velocities and temperatures back to the equilibrium values U^+ and T^+. For the generalized shock structure we describe, it is sufficient that these equations imply $U_p \to U_f$ and $T_p \to T_f$ as $\xi \to \infty$.

The jump conditions (5.35), (5.37), and (5.39) show that for a generalized gas, the generalized shock relations are precisely those for a gas with equivalent properties. The shock thickness if $O(\varepsilon)$.

6. SMALL DISTRUBANCE THEORY

With the derivation of the generalized gas model (4.23)-(4.25) and generalized shock conditions (5.35), (5.37), and (5.39), we have achieved considerable simplification of the original problem. The dusty gas may be regarded as an equivalent gas with a modified value of γ. Assuming that we have potential flow, it is therefore possible to formulate a small-disturbance theory for transonic thin airfoils in dusty gases, analogous to that in [2]. The transonic similarity parameter for this case is a function of the new value of γ. It will be shown how this modified similarity parameter is related to the usual one for the case of no dust, so that a method of estimating the effect of the dust on the usual small-disturbance theory may be achieved.

We formulate the boundary value problem for a thin airfoil, in a dusty gas and travelling in the transonic range, as follows

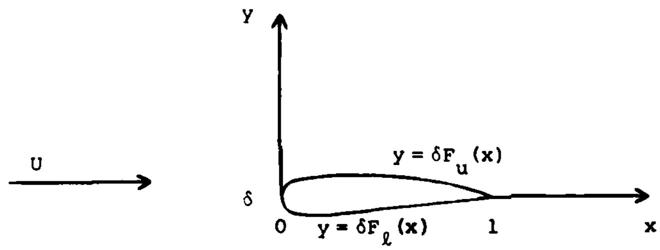


Figure 6-1

As shown in Figure 6-1, the free-stream velocity is U (in the x direction only), where the coordinates x, y have been normalized with respect to the airfoil chord. The function F which defines the airfoil surface satisfies $\max\limits_{x\in[0,1]} |F_u(x) - F_\ell(x)| = 1.$

We assume that a potential Φ exists, so that the flow components are given by

$$\nabla\Phi = (U + u,v) = U\vec{e}_x + \vec{q} \qquad (6.1)$$

where $u,v,$ represent the distrubance velocities, and $\vec{e}_x$ is the unit vector in the x direction.

The potential ϕ satisfies the system

$$(\hat{a} - \phi_x^2)\phi_{xx} - 2\phi_x\phi_y\phi_{xy} + (\hat{a}^2 - \phi_y^2)\phi_{yy} = 0 , \qquad (6.2)$$

$$\frac{\hat{a}^2}{\hat{\gamma} - 1} + \frac{q^2}{2} = \frac{\hat{a}_\infty^2}{\hat{\gamma} - 1} + \frac{U^2}{2} . \qquad (6.3)$$

Here, $\hat{a}$ is the local speed of sound in the dusty gas, which we are going to distinguish from a, the sound speed in the gas without dust.

ϕ satisfies the boundary condition

$$\frac{\phi_y(x,\delta F_{u,\ell}(x))}{\phi_x(x,\delta F_{u,\ell}(x))} = \delta F'_{u,\ell}(x) \qquad (6.4)$$

of tangent flow along the wing, and

$$\phi \to U_x \quad \text{as} \quad x \to -\infty , \qquad (6.5)$$

corresponding to uniform flow for upstream. We also assume that the Kutta condition holds: the flow leaves the trailing edge smoothly.

Under the condition of small distrubances, we may derive a simpler, approximate equation for the problem. This derivation is achieved through a limit process expansion for ϕ, based on the limit process $\delta \to 0$, with $x, \tilde{y} = \delta^{1/3} y$, and $\hat{K} = \frac{1 - \hat{M}_\infty^2}{\delta^{2/3}}$ fixed as $\hat{M}_\infty \to 1$. It has the form

$$\Phi(x,y;\delta) = U\{x + \delta^{2/3}\phi(x,\tilde{y}) + \ldots\} . \qquad (6.6)$$

Then ϕ satisfies

$$(\hat{K} - (\hat{\gamma} + 1)\phi_x)\phi_{xx} + \phi_{\tilde{y}\tilde{y}} = 0 \qquad (6.7)$$

and the boundary conditions

$$\phi_{\tilde{y}}(x,0^{\pm}) = F'_{u,\ell}(x) \qquad 0 < x < 1 \qquad (6.8)$$

$$\phi_x, \phi_{\tilde{y}} \to 0 \quad \text{as} \quad x \to -\infty . \qquad (6.9)$$

In this context,the Kutta condition may be written as

$$[\phi_x]_{TE} = 0 , \qquad (6.10)$$

where TE means trailing edge.

Additionally, shock jump conditions must be imposed in order to have a complete problem for ϕ. The conditions may be derived from the continuity of the transonic mass flux vector

$$\rho\vec{q} = \rho_\infty U\{\vec{e}_x(1 + \delta^{4/3}(\hat{K}\phi_x - \frac{\hat{\gamma}+1}{2}\phi_x^2) + \dots) + \delta\phi_{\tilde{y}}\} + \dots \quad (6.11)$$

If the shock surface is given by

$$S(x,\tilde{y}) = x - g(\tilde{y}) = 0 ,$$

the jump condition is

$$[\hat{K}\phi_x - \frac{\hat{\gamma}+1}{2}\phi_x^2]_s - [\phi_{\tilde{y}}]_s g'(\tilde{y}) = 0 . \quad (6.12)$$

Furthermore, there is no jump in the tangential component of velocity across the shock, which may be guaranteed by imposing

$$[\phi]_s = 0 . \quad (6.13)$$

The form of (6.7) shows that for two flows at different transonic Mach numbers, but with the same values of $\hat{K}$, these two flows will be geometrically similar (the difference will be that the 'size' of the disturbance will be determined by different factors of $\delta^{2/3}$). An analogous similarity law holds for the gas alone, governed by the similarity parameter K. Both of these similarity laws relate families of flows, for different values of the displacement thickness at a corresponding Mach number.

We wish to see how the change in the ratio of the specific heats for the dusty gas, $\hat{\gamma}$, affects the flow. This will be done by finding a correspondence between the similarity parameters in the two cases.

We shall develop the correspondence by transforming the boundary value problem (6.2)-(6.9) for the dusty gas into a problem for the gas without dust.

Let $\psi(\hat{K}) = \phi$ be the solution operator for the dusty gas boundary value problem. By defining

$$\bar{K} = \omega^{-2}\hat{K} \quad (6.14a)$$

$$\bar{y} = \omega\tilde{y} = \bar{\delta}^{1/3}y \quad (6.14b)$$

$$\bar{\phi} = \omega\phi \quad (6.14c)$$

where $\omega = (\frac{\hat{\gamma}+1}{\gamma+1})^{1/3}$, and $\bar{\delta} = \omega^3\delta$, we see that (6.7), (6.8) become

$$(\bar{K} - (\gamma + 1)\bar{\phi}_x)\bar{\phi}_{xx} + \bar{\phi}_{\bar{y}\bar{y}} = 0 \,, \tag{6.15}$$

$$\bar{\phi}_{\bar{y}}(x,0^{\pm}) = F'_{u,\ell}(x), \qquad 0 < x < 1 \,. \tag{6.16}$$

The other boundary conditions, (6.9) and (6.10), transform readily in the correct manner. The shock jump condition (6.12), since it can be derived from the divergence form of (6.7), imposes no additional restriction upon transformation by (6.14). This is also true for (6.13).

Therefore, under (6.14) the entire problem for $\phi(x,\tilde{y};\hat{\kappa})$ transforms to the corresponding problem for $\bar{\phi}(x,\bar{y};\bar{\kappa})$, which is the case of the gas without dust. Comparing (6.7) and (6.15), we conclude

$$\psi(\hat{K}) = \omega\psi(\bar{K}) \,. \tag{6.17}$$

Now, the similarity parameter $\bar{K}$ for the new problem is of the form

$$\bar{K} = \frac{1 - \bar{M}_\infty^2}{\bar{\delta}^{2/3}} = \frac{1 - \bar{M}_\infty^2}{(\omega^3\delta)^{2/3}} = \frac{1 - \bar{M}_\infty^2}{\omega^2\delta^{2/3}} = \omega^{-2}\hat{\kappa} \,, \tag{6.18}$$

from which we conclude $\bar{M}_\infty = \hat{M}_\infty$. Combining (6.17) and (6.18) gives

$$\psi(\hat{K}) = \omega\psi(\omega^{-2}\bar{\kappa}) \,. \tag{6.19}$$

This relation is equivalent to

$$\phi(x,\tilde{y};\hat{\kappa}) = \omega\bar{\phi}(x,\bar{y};\omega^{-2}\bar{\kappa}) \,. \tag{6.20}$$

To complete the details, we note

$$\hat{K} = \frac{1 - \hat{M}_\infty^2}{\delta^{2/3}} \quad \text{where} \quad \hat{a}_\infty^2 = \frac{\hat{\gamma}}{\gamma\left(1 + \frac{\sigma_\ell^{(0)}}{\rho_\ell^{(0)}}\right)} a_\infty^2$$

from Section 4, so that

$$\hat{K} = \frac{1 - \frac{\gamma\left(1 + \frac{\sigma_\ell^{(0)}}{\rho_\ell^{(0)}}\right)}{\hat{\gamma}} M_\infty^2}{\delta^{2/3}} \,, \tag{6.21}$$

where $\hat{\gamma}$ is given by (4.21).

Thus, we see from (6.20) that the small disturbance problem for a dusty gas is equivalent to one for a clear gas, with a modified amplitude of disturbances, a changed wing thickness ratio and similarity parameter.

The transformation parameter ω, as a function of $\alpha^{(0)}f/\rho_f^{(0)}$, is shown in Figure 6.2 for $c/(\hat{c} + r) = 1.0$, which is a representative value for many materials.

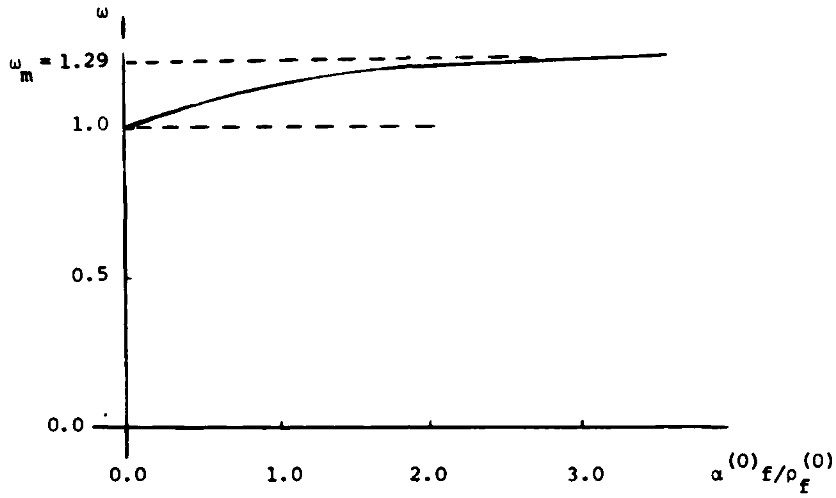


Figure 6.2. ω vs. $\alpha^{(0)}f/\rho_f^{(0)}$

Thus, for example, a dusty flow with a mass loading of 2.0 corresponds to a value of ω of 1.21, which means that if the airfoil has thickness δ, the flow is equivalent to that of a clear gas around an airfoil of the same shape, but of thickness $\bar{\delta} = \omega^3\delta = 1.77\delta$. The equivalent similarity parameter $\bar{K}$ is $\bar{K} = \omega^{-2}\hat{K} = 0.68\hat{K}$. This implies that for a given airfoil, of thickness δ_0, the solution in a clear gas for thickness $\delta_1 = \omega^3\delta_0$ at similarity parameter K_1 can be obtained from the solution in a dusty gas with a value of ω equal to $\omega = (\delta_1/\delta_0)^{1/3}$, and similarity parameter $K_0 = \omega^2K_1$. The loading $\alpha^{(0)}f/\rho_f^{(0)}$, can be obtained from

$$\omega = (\delta_1/\delta_0)^{1/3} = \left(\frac{\hat{\gamma} + 1}{\gamma + 1}\right)^{1/3}$$

so that

$$\hat{\gamma} = -1 + \frac{\delta_1}{\delta_0}(\gamma + 1)$$

But

$$\hat{\gamma} = \frac{\gamma + \dfrac{\alpha^{(0)}f}{\rho_f^{(0)}} \dfrac{c}{\hat{c} + r}}{1 + \dfrac{\alpha^{(0)}f}{\rho_f^{(0)}} \dfrac{c}{\hat{c} + r}} = -1 + \frac{\delta_1}{\delta_0}(\gamma + 1)$$

Thus,

$$\frac{\alpha^{(0)}f}{\rho_f^{(0)}} = \frac{(\gamma + 1)\left(\dfrac{\delta_1}{\delta_0} - 1\right)}{2 - \dfrac{\delta_1}{\delta_0}(\gamma + 1)} \cdot \frac{\hat{c} + r}{c} .$$

REFERENCES

1. Ishii, M., 1975, Thermo-fluid dynamic theory of two-phase flow, Eyrolles, Paris, France.

2. J. Kevorkian and J. D. Cole, Perturbation Methods in Applied Mathematics, Chapter 5, Springer-Verlag, 1981.

3. Bolz, R. E. and Tuve, G. L., eds., 1973, Handbook of tables for applied engineering science, 2nd ed., Chemical Rubber Co., Cleveland, OH.

DAD/FJZ/ed

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 2521	2. GOVT ACCESSION NO. AD-A130 511	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) TRANSONIC SMALL-DISTRUBANCE THEORY FOR DUSTY GASES		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Donald A. Drew and Fredrick J. Zeigler		8. CONTRACT OR GRANT NUMBER(s) DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 2 - Physical Mathematics
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709		12. REPORT DATE May 1983
		13. NUMBER OF PAGES 22
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Two-phase flow, dusty gas, transonic small-disturbance, similarity

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The correction to transonic small-disturbance theory induced by the addition of contaminant particles (e.g., dust) to the gas is analyzed. Two-phase flow equations governing the particles and inviscid fluid are used. The dusty gas version, a particular limit of the flow equations in which the volumetric concentration of the particles is small, but the mass

(cont.)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ABSTRACT (cont.)

concentrations of particles and fluid are comparable, is employed. In this setting, a model in which the gas and particles may be viewed as an equivalent gas with modified properties, is derived. This "generalized gas" model behaves like a normal gas with a modified value of γ (the ratio of specific heats). Using this model, a simple method of analyzing transonic small-disturbance theory, by employing a modified transonic similarity parameter, is used to account for the effect of the dust.

END

FILMED

8-83

DTIC